QUASISTATIONARY FLOW OF A REACTING FLUID, LOSING FLUIDITY WITH HIGH DEGREES OF TRANSFORMATION

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Due to velocity differences in the flow of a reacting fluid, the degree of transformation at the walls can greatly exceed the average value over the cross section, and if the reacting fluid loses fluidity with high degrees of transformation, then a continuous growth of a congealed layer of the reaction products occurs at the walls. Thus, in the case of polymerization at high monomer concentrations (or in the bulk), reactor channels and pipelines are observed to be overgrown with the polymer. This phenomenon is well known to engineers, but theoretical investigations of the dynamics of this process have not been carried out. In this paper, the dynamics of the process will be examined under the following assumptions: The temperature of the reacting fluid remains constant, the fluid is a Newtonian fluid, the flow is laminar, and the process is quasistationary. Relations are obtained that permit forecasting the rate of growth of the stationary layer of reaction products. In addition to the hydrodynamic characteristics, the rate of growth depends also only on a single parameter, which is determined by the nature of the change in the viscosity with time and is found from a solution of a self-similar boundary-value problem.

1. We are examining the isothermal flow of a reacting Newtonian fluid, which losses fluidity at high degrees of transformation. The fluid is assumed to be quite viscous, while the change in the radius of the flow along the pipe (of a flow-through tubular reactor) is smooth enough that in each separate section, the fluid flow can be assumed to be practically plane-parallel. This approximation is widely used in theoretical investigations of different problems concerning the flow of a fluid with variable properties (see, for example, [1-3]); in order to find the flow velocity in this approximation, it follows from the general equations of motion of a Newtonian fluid [4] that

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial v}{\partial r}\right)+\frac{\partial p}{\partial z}=0,\quad 0\leqslant r\leqslant R(z,t),\ 0<z\leqslant L, \tag{1.1}$$

where v is the axial component of the flow velocity; μ, viscosity of the fluid; r, distance from the axis of the pipe; p = p(z, t), difference between the pressure at the inlet to the tube and the pressure at the given section; z, distance from the beginning of the pipe; R, radius of the flow (inner radius of the stationary layer of reaction products); L, pipe length; and t, time. We shall assume that the density of the fluid ρ is constant and the radial component of the velocity w is obtained from the equation of continuity

$$\frac{1}{r}\frac{\partial}{\partial r}(rw)+\frac{\partial v}{\partial z}=0. \tag{1.2}$$

In view of the smallness of the diffusion coefficients in fluids, the effect of diffusion on the flow of the reaction in the flow can be neglected. In this connection, at any point of the flow of the reacting isothermal fluid, the depth of transformation and the mechanical properties of the fluid depend only on the time, during which the given element of fluid is already found in the flow (flow-through reactor). The fluid motion has no effect on these dependences and they can be assumed to be fixed.

The loss of fluidity formally means that after the passage of some period of time t_0, determined by the rate of the chemical transformations, the viscosity of the fluid becomes infinite. In accordance with this, we shall assume a time dependence of the viscosity in the form

$$\mu_0/\mu = f(\vartheta), \tag{1.3}$$

where μ_0 is the value of the viscosity, corresponding to the initial fluid; ϑ is the dimensionless time, passing from the time that the element of the fluid enters the reactor. As ϑ increases, the function $f(\vartheta)$ decreases from the value f = 1 at $\vartheta = 0$, $f(\vartheta) = 0$ for $\vartheta > 1$. For $\vartheta < 1$, $f(\vartheta) > 0$.

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The rate of growth of the stationary layer of reaction products is determined by the condition that the time over which the fluid elements reach the surface of this layer equals t_0. The distribution $\vartheta = \vartheta(t, z, r)$ must thus be such that at the periphery of the flow the condition $\vartheta = 1$ be satisfied. At the beginning of the pipe (reactor), $\vartheta = 0$, and then the value of ϑ increases with distance downstream along the flow. For a fixed fluid element, the increment to ϑ is related to its displacement and velocity along the flow by the relation $d\vartheta = dz/vt_0$.

Compared to the average residence time of the fluid in the reactor, the period t_0 is assumed to be sufficiently long that the change in the properties of the fluid affect only a narrow layer moving at the edges of the flow and appreciable changes in the flow radius occur only over a time greatly exceeding t_0. The flow characteristics and their rate of change with time, in this case, will be completely determined by the running distribution of the radius of the flow along the pipe and for this reason it is appropriate to call the given approximation a quasistationary approximation.

The approximation indicated means that

$$U/Qt_0 \ll 1, \tag{1.4}$$

where Q is the volume flow rate of the fluid; U is the volume of the channel, free of the stationary layer of reaction products:

$$U(z, t) = \int_0^z \pi R^2(\zeta, t)\, d\zeta. \tag{1.5}$$

Condition (1.4), in any case, is satisfied near the beginning of the pipe. And, since the free volume of the channel U decreases with growth of the stationary layer of reaction products, even if condition (1.4) is not initially satisfied for part of the pipe, with time and with the same fluid flow rate, the required degree of smallness of the quantity U/Qt_0 will be attained over the entire length of pipe. The effect of the preceding nonquasistationary stage could be taken into account by giving an appropriate distribution of the flow radius.

From the mathematical point of view, the quasistationary approximation being examined corresponds to the leading term in the asymptotic expression for $U/Qt_0 \to 0$.

2. We shall first examine a model example, in which the viscosity of the fluid remains unchanged throughout the period t_0, $f(\vartheta) = 1$ for all $\vartheta < 1$. The solution of the problem in this case is greatly simplified, since it is no longer necessary to examine the hydrodynamic part. The flow velocity in each separate section is described by a parabolic Poiseuille profile

$$v = (2Q/\pi R^2)(1 - r^2/R^2), \quad 0 \leqslant r \leqslant R(z, t). \tag{2.1}$$

We shall denote by $q = q(t, z, r)$ the amount of fluid, crossing the section $z = \text{const}$ at a given time at distances from the axis greater than a given value r

$$q = \int_r^R v 2\pi r dr, \tag{2.2}$$

and we shall follow the value of q, corresponding to a fixed fluid element. Since from (2.2), taking into account (1.2), it follows that

$$\partial q/\partial r = -2\pi r v, \quad \partial q/\partial z = 2\pi r w. \tag{2.3}$$

We have for the change in the value of q corresponding to the fixed element

$$dq/dt \equiv \partial q/\partial t + v\partial q/\partial z + w\partial q/\partial r = \partial q/\partial t. \tag{2.4}$$

Substitution of (2.1) into (2.2) shows that in this case

$$q = Q(1 - r^2/R^2)^2, \tag{2.5}$$

from which it follows, taking into account (2.1), that

$$\partial q/\partial t = v\pi(\partial R^2/\partial t)r^2/R^2, \tag{2.6}$$

while, since within the scope of the quasistationary approximation it is sufficient to limit the analysis to fluid elements moving at the edges of the flow, in (2.6), we can set $r^2/R^2 \simeq 1$. As a result, substituting (2.6) into (2.4) and integrating along the trajectory of motion, we have

$$q = q_0 + \int_0^z \pi \frac{\partial R^2}{\partial t} dz, \tag{2.7}$$

where q_0 is the value of the quantity q at the pipe inlet corresponding to the given fluid element, while the values of $\partial R^2(z, t)/\partial t$ in calculating the integral are taken for those times at which the fluid element being examined intersects the given section. But, according to the assumptions made, the changes in the quantities occurring over time intervals of the order of t_0 can be neglected. For this reason, in (2.7), all quantities can be assumed to relate to the same time and, changing the order of the integration and differentiation operations, relation (2.7) taking into account (1.5) can be represented in the form

$$q = q_0 + U_t'(z, t). \tag{2.8}$$

Relation (2.8) already permits relating the parameters of the trajectory of a separate fluid element to the rate of growth of the stationary layer of reaction products. Thus, if the fluid element being examined reaches the surface of the stationary layer of products at a distance z_0 from the beginning of the pipe, then, determining the value of q_0 corresponding to this element from the condition that on the surface of the stationary layer $q = 0$, from (2.8) we obtain for its trajectory of motion

$$q = U_t'(z, t) - U_t'(z_0, t). \tag{2.9}$$

The velocity is determined by the relation $v = 2(qQ^{1/2})/\pi R^2$, which follows from (2.1) and (2.2); in order to calculate the value of the dimensionless time $\vartheta(z, z_0)$, over which the fluid element being examined reaches the given section, taking into account (1.5), we have

$$d\vartheta \equiv \frac{dz}{vt_0} = \frac{1}{2t_0\sqrt{Q}} \frac{U_z'(z, t)\, dz}{\sqrt{U_t'(z, t) - U_t'(z_0, t)}}. \tag{2.10}$$

At $z = z_0$, the condition $\vartheta = 1$ must be satisfied, and since the fluid element being examined, and thus the values of z_0, are chosen arbitrarily, an integral equation for $\partial U/\partial t$, i.e., the rate of growth of the stationary layer of reaction products, follows in an obvious manner from (2.10). It may be verified by direct substitution that the solution of this equation is

$$\frac{\partial U}{\partial t} = -\frac{\pi^2}{16}\frac{U^2}{Qt_0^2}. \tag{2.11}$$

Indeed, substituting (2.11) into (2.10) and integrating, we find for the change in the value of ϑ along the trajectory of motion of the element, reaching at $z = z_0$ the surface of the stationary layer of reaction products,

$$\vartheta(z, z_0) = (2/\pi) \arcsin (U/U_0), \quad U_0 \equiv U(z_0, t), \tag{2.12}$$

from where it indeed follows that at $z = z_0$ $(U = U_0)$, $\vartheta = 1$. Eliminating from (2.12) with the help of (2.11) and (2.9), the value of U_0, we finally obtain the following expression for the distribution of ϑ:

$$\vartheta = \frac{2}{\pi} \arcsin \left[\frac{\pi}{4} \Big/ \sqrt{\frac{\pi^2}{16} + \frac{q}{Q}\left(\frac{Qt_0}{U}\right)^2} \right]. \tag{2.13}$$

Thus, the value of ϑ depends only on a single unique complex of variables. We also note that, applying relation (2.11) and (2.13), we can check the validity of the order of smallness of various quantities for $U/Qt_0 \to 0$, used in the calculations carried out above.

3. We shall now proceed to examine the general case of an arbitrary dependence (1.3) of the fluid viscosity on the residence time in the reactor. In this case, in accordance with the solution of the model problem obtained above, we shall seek the distribution of the residence time of fluid elements in the flow in the form of a function of the self-similar variable

$$X = (q/Q)(Qt_0/U)^2, \tag{3.1}$$

while for the rate of growth of the stationary layer of reaction products, in analogy to (2.11), we shall assume the relation

$$\partial U/\partial t = -a^2(U/t_0)U/Qt_0, \tag{3.2}$$

where the constant a must be determined in solving the problem; differentiating (3.2) with respec to z and taking into account (1.5), the following expression is obtained for the time rate of change of the channel radius R:

$$\partial R^2/\partial t = -2a^2(R^2/t_0)U/Qt_0. \tag{3.3}$$

The self-similar equation sought is a result of the obvious identity $t_0 d\vartheta/dt \equiv 1$, which in this case takes the form

$$t_0(d\vartheta/dX)dX/dt = 1, \tag{3.4}$$

while the value of a must be such that a solution $\vartheta = \vartheta(x)$ of the equation following from (3.4) by substituting into the expression for dX/dt (it is understood, naturally, that for $U/Qt_0 \to 0$, the total derivative dX/dt will be a function only of the variable X), satisfying the boundary conditions

$$\vartheta = 1 \text{ for } X = 0, \ \vartheta \to 0 \text{ for } X \to \infty \tag{3.5}$$

exists.

Since it follows from (3.1), taking into account (2.4) and (1.5) that

$$\frac{dX}{dt} = \frac{\partial X}{\partial t} + v\frac{\partial X}{\partial z} + w\frac{\partial X}{\partial r} = \frac{\partial X}{\partial t} - 2v\frac{\pi R^2}{U}X, \tag{3.6}$$

the calculation of the total derivative dx/dt reduces to finding expressions for $\partial X/\partial t$ and v. The basis for finding the flow velocity v is the equation, following from (1.1),

$$\mu\partial v/\partial r = -(1/2)r\partial p/\partial z. \tag{3.7}$$

In accordance with (2.3) and (3.1), we have

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial q}\frac{\partial q}{\partial r} = -2\pi r v\frac{\partial v}{\partial q} = -\pi r\frac{Qt_0^2}{U^2}\frac{\partial v^2}{\partial X}, \tag{3.8}$$

while since for $U/Qt_0 \ll 1$, the properties of the fluid change only in a narrow layer at the edges of the flow, for $\partial p/\partial z$ in the limiting case being examined, in accordance with Poiseuille's law [4], we have

$$\partial p/\partial z = 8\mu_0 Q/\pi R^4. \tag{3.9}$$

As a result, substituting (3.8), (3.9), and (1.3) into (3.7) and integrating, we obtain for the flow velocity

$$v = \frac{2U}{\pi R^2 t_0}\sqrt{F(X)}, \quad F(X) = \int_0^X f(\vartheta(x))\,dx. \tag{3.10}$$

In order to find the partial derivative $\partial X/\partial t$, we shall proceed as follows: We shall determine the dependence $r = r(t, z, X)$, and then we shall differentiate the expression obtained with respect to t with fixed z and X. According to (2.3), $\partial r^2/\partial q = -1/\pi v$, while substituting here (3.10) and integrating, taking into account (3.1), we have

$$1 - \frac{r^2}{R^2} = \frac{1}{2}\frac{U}{Qt_0}\int_0^X \frac{dx}{\sqrt{F(x)}}. \tag{3.11}$$

For the partial derivative $\partial X/\partial t$, it follows from (3.11) taking into account (3.2) and (3.3),

$$\frac{1}{\sqrt{F(X)}}\frac{\partial X}{\partial t}\Big| = -4a^2 + \left(\frac{t_0}{Q}\frac{\partial Q}{\partial t} + 3a^2\frac{U}{Qt_0}\right)\int_0^X \frac{dx}{\sqrt{F(x)}}. \tag{3.12}$$

Assuming that the change of the fluid flow rate with time, if it occurs, occurs sufficiently slowly, we obtain from (3.12) for $U/Qt_0 \to 0$

$$t_0\partial X/\partial t = -4a^2\sqrt{F(X)}. \tag{3.13}$$

Substituting (3.13) and (3.10) into (3.6) indeed shows that for $U/Qt_0 \to 0$ the total derivative dX/dt is a function of only the single self-similar variable X

$$t_0 dX/dt = -4(X + a^2)\sqrt{F(X)}, \tag{3.14}$$

and, thus, the following self-similar integrodifferential equation follows from (3.4) and (3.14):

TABLE 1

n	a	t_*/t_0
10	0,86	0,92
5	0,92	0,86
2	1,08	0,73
1	1,30	0,60
1/2	1,65	0,47
1/4	2,20	0,36
1/8	3,00	0,26

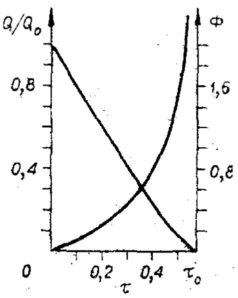

Fig. 1

$$\frac{dX}{d\vartheta} = -4(X+a^2)\left[\int_0^X f(\vartheta(x))\,dx\right]^{1/2}. \tag{3.15}$$

Differentiating (3.15) and eliminating the value of the integral, we obtain a nonlinear differential equation, which assumes the following form if ϑ is taken as the independent variable:

$$Y d^2Y/d\vartheta^2 = (dY/d\vartheta)^2 + 8Y^3 f(\vartheta), \quad Y \equiv X + a^2. \tag{3.16}$$

The solution of Eq. (3.16) must satisfy the boundary conditions

$$Y \to \infty \text{ for } \vartheta \to 0, \; Y = a^2, \; dY/d\vartheta = 0 \text{ for } \vartheta = 1. \tag{3.17}$$

These conditions, of which the first two correspond to (3.5), while the last condition follows from (3.15) for $X \to 0$, completely determine the solution and value of a sought.

In analyzing the boundary-value problem obtained, it is convenient to introduce the variable $y = \frac{1}{2}\sqrt{Y}$, which in contrast to Y remains bounded for all ϑ. Transforming to the new variable, we obtain the following equation from (3.16):

$$y d^2y/d\vartheta^2 = (dy/d\vartheta)^2 - f(\vartheta), \tag{3.18}$$

whose solution must satisfy the conditions

$$y = 0 \text{ for } \vartheta = 0, \; dy/d\vartheta = 0 \text{ for } \vartheta = 1. \tag{3.19}$$

Let us assume that conditions (3.19) are satisfied by two solutions $y_1(\vartheta)$ and $y_2(\vartheta)$ of Eq. (3.18) such that $y_1(1) > y_2(1)$. Since it follows from (3.18) that $y'(0) = [f(0)]^{1/2}$ for these curves, for $\vartheta \to 0$, we have $y_1(\vartheta)/y_2(\vartheta) \to y_1'(0)/y_2'(0) = 1$. At the same time, since Eq. (3.18) can be represented also in the form $d^2 \ln y/d\vartheta^2 = -y^{-2} f(\vartheta)$, we obtain the following expression for the ratio y_1/y_2:

$$\frac{d}{d\vartheta}\left|\ln\frac{y_1(\vartheta)}{y_2(\vartheta)}\right| = -\int_\vartheta^1 |y_1^{-2} - y_2^{-2}|\, f d\vartheta \leqslant 0,$$

from where it follows that $y_1(\vartheta)/y_2(\vartheta) \geq y_1(1)/y_2(1) > 1$, which contradicts the limiting case found earlier for $\vartheta \to 0$. The contradiction indicated proves the uniqueness of the solution.

The solution of the problem can be found numerically by the ranging method, examining the different integral curves of Eq. (3.18) satisfying condition (3.19) with $\vartheta = 1$. Integral curves for which $y(0) > 0$ correspond to the values $y(1) > \frac{1}{2}a$; if, on the other hand, the curve reaches the $y = 0$ axis for $\vartheta > 0$, then $y(1) < \frac{1}{2}a$.

Table 1 presents the values of the parameter a obtained numerically for the case $f(\vartheta) = 1 - \vartheta^n$. In addition, the ratio $t_*/t \equiv (\pi/4)/a$ is indicated. The quantity t_* introduced here represents the characteristic period of time for each fixed law of variation of viscosity. The smaller n, i.e., the earlier viscosity of the fluid, begins to differ appreciably from the initial value, the smaller is the value of t_*. As n increases, the function $f(\vartheta)$ approaches the case of the jumplike change in viscosity examined in the preceding section, in which $a = \pi/4$, and the quantity t_* approaches t_0.

4. Thus, aside from the hydrodynamic characteristics, the only parameter that determines the rate of growth of the stationary layer of reaction products is the quantity

$$\frac{\partial U}{\partial t} = -\frac{\pi^2}{16}\frac{U^2}{Qt_*^2}, \tag{4.1}$$

while integrating (4.1), after simple transformations, we obtain the following for the change in the free volume U channel radius R with time:

$$R(z, 0)/R(z, t) = U(z, 0)/U(z, t) = 1 + G(t)U(z, 0), \tag{4.2}$$

where the function G(t) is determined by the condition

$$dG/dt = \pi^2/16Qt_*^2, \quad G(0) = 0. \tag{4.3}$$

For any finite value of G, the channel radius differs from 0 and fluid motion is possible.

The relations obtained completely determine the behavior of the system. Thus, if initially there is no congealed layer of reaction products at the walls of the tube $[R(z, 0) \equiv R_0]$, then it follows for the radius of the flow from (4.2)

$$R_0/R = 1 + \Phi(t)\, z/L, \quad \Phi(t) \equiv G(t)\,\pi R_0^2 L, \tag{4.4}$$

while, for the pressure drop, we obtain from (3.9) and (4.4)

$$P = \frac{8\mu_0 Q}{\pi}\int_0^L \frac{dz}{R^4(z, t)} = \frac{8\mu_0 LQ}{\pi R_0^4}\frac{(\Phi+1)^5 - 1}{5\Phi}. \tag{4.5}$$

In the case of a constant pressure drop, for the fluid flow rate, it follows from (4.5)

$$\frac{Q}{Q_0} \equiv \frac{5\Phi}{(1+\Phi)^5 - 1}, \quad Q_0 \equiv Q(0) = \frac{\pi R_0^4 P}{8\mu_0 L}, \tag{4.6}$$

and for finding the dependence $\Phi(t)$ we have from (4.3) and (4.6)

$$\tau = \frac{\pi^2}{16}\frac{\pi R_0^2 L}{Qt_*^2}t = \int_0^\Phi \frac{5\varphi\, d\varphi}{(1+\varphi)^5 - 1}. \tag{4.7}$$

The dependences $\Phi(\tau)$ and $Q(\tau)$ following from (4.7) and (4.6) are shown in the figure, from where it is evident that over most of the range of variation, the fluid flow rate is satisfactorily approximated by the first two terms in a Taylor series expansion, $Q/Q_0 \simeq 1 - 2\tau$. The motion of the fluid ceases for

$$\tau_0 = \int_0^\infty \frac{5\varphi\, d\varphi}{(1+\varphi)^5 - 1} \simeq 0{,}5756.$$

If, on the other hand, the flow rate of the fluid is maintained constant, then $\Phi = \tau$ and from (4.5) we obtain the following for the pressure drop:

$$\frac{P}{P_0} = \frac{(1+\tau)^5 - 1}{5\tau} \quad P_0 \equiv P(0) = \frac{8\mu_0 QL}{\pi R_0^4},$$

while from (4.4), we obtain the following for the change in the channel radius with time:

$$\frac{R_0}{R} = 1 + \frac{\pi^2}{16}\frac{\pi R_0^2 z}{Qt_*^2}t. \tag{4.8}$$

The linearity of the change in the ratio R_0/R permits determining empirically the quantity t_* from experiments with constant fluid flow rate.

In conclusion, we point out that if together with the change in the viscosity, the fluid density also changes from the initial value ρ_0 to ρ_1, then instead of (4.1) we shall have

$$\frac{\partial U}{\partial t} = -\frac{\pi^2}{16}\frac{\rho_0}{\rho_1}\frac{U^2}{Qt_*^2} \tag{4.9}$$

and an addition corresponding to (4.9) appears in expressions (4.3), (4.7), and (4.8). The quantity t_* in this case, as before, is determined by the relation $t_*/t_0 \equiv (4/\pi)/a$ and the function on $f(\vartheta) = \mu_0\rho_0/\rho\mu$ will appear only in Eqs. (3.16) and (3.18), which are used to find the parameter a (or t_*), instead of (1.3).

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EFFECTIVE PERMEABILITY OF A HIGHLY POROUS MEDIUM

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The problem of the effective conductivity of a medium with a low concentration of inclusions has been treated in many papers (e.g., [1]). The case of a medium with a random distribution of circular inclusions characterized by a binary correlation function was treated in [2] by using the apparatus of ensemble averages. We use methods of the theory of functions of a complex variable to solve the two-dimensional problem of the effective permeability of a medium with translational symmetry of an arrangement of circular inclusions. Since a correlation function does not have to be defined for an ordered arrangement of inclusions, the effective permeability of the medium can be determined when the concentration of inclusions is not low. By using methods of the theory of functions of a complex variable, we obtain an effective solution of this kind of problem for inclusions of arbitrary shape by conformal mapping onto the exterior of a unit circle. In this sense the solution of the basic problem is reduced. The problem was solved by using the approach developed in [3, 4] for determining the state of stress of a plane weakened by an infinite number of circular holes. The basic idea of this approach consists in representing the required solution in the form of a Laurent series by expanding it in terms of the small parameter $\varepsilon = 1/l$, where l is the distance between centers of the inclusions, and using the basic idea of the Bubnov–Galerkin method to find the expansion coefficients. As in the elasticity problem, this is an effective method of solving transmissibility problems in a medium with an infinite number of inclusions. By averaging the solution over a macroscopic volume the effective transmissibility coefficient of such a medium can be determined.

Filtration in a Medium with Circular Inclusions. Let us consider the steady filtration of a fluid in a medium with circular inclusions arranged as shown in Fig. 1. Without loss of generality, we take the inclusions of unit radius. The distances along the x and y axes between the centers of neighboring circles are assumed equal to l. Thus, the centers of the circles lie at the points

$$z_{n,p} = l(n + ip),$$

where $i = \sqrt{-1}$; $n = 0, \pm1, \pm2, \ldots, \pm\infty$; $p = 0, \pm2, \ldots \pm\infty$. As in [5], it is convenient to describe filtration flow by introducing the complex potentials

$$\varphi_\nu = (k_\nu/\mu)P_\nu + i\psi_\nu,\ \nu = 0,\ 1.$$

Here φ_0 corresponds to the filtration region in the medium outside an inclusion, and φ_1 to the region inside an inclusion; the k_ν are the permeabilities of the medium and inclusion, respectively; μ is the viscosity of the fluid; the P_ν are the pressures of the fluid in the medium and within an inclusion respectively; the ψ_ν are the flow functions. The complex potentials must satisfy Laplace's equation

$$\Delta\varphi_\nu = 0 \tag{1}$$

and are analytic in the respective domains of definition. In addition, the joining conditions

$$\frac{\partial}{\partial n_1}\operatorname{Re}\varphi_0 = \frac{\partial}{\partial n_1}\operatorname{Re}\varphi_1; \tag{2}$$

$$\frac{\partial}{\partial s}\operatorname{Re}\frac{1}{k_0}\varphi_0 = \frac{\partial}{\partial s}\operatorname{Re}\frac{1}{k_1}\varphi_1. \tag{3}$$

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